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LOW-STRESS STATE IN AN INHOMOGENEOUS COMPOUND WEDGE WITH MIXED BOUNDARY CONDITIONS

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A study has been made on the effects of inhomogeneity on the low-stress state at the edge of the contact surface in a compound wedge having power-law hardening under conditions of longitudinal shear and planar strain. It is assumed that one face of the wedge is free and the other is rigidly gripped. A solution has been given to an analogous treatment for a homogeneous compound wedge in [1]. The low-stress state has been examined for linearly elastic piecewise-homogeneous bodies in [2, 3]. In [4], low-stress states have been considered for an inhomogeneous compound wedge with free faces on longitudinal shear and planar strain.

Here I derive the conditions for restricted stress at the tip of an inhomogeneous compound wedge. It is shown that the low-stress zones are related to the inhomogeneity in the mechanical parameters.

1. Longitudinal Shear. Consider two long cylindrical bodies composed of inhomogeneous materials showing power-law hardening, which are joined together over a certain part of the side surfaces with complete adhesion. The angular point at the edge of the contact surface is under the conditions of longitudinal shear. One face in the angular part of the body is rigidly gripped. We place the origin of a cylindrical coordinate system at the angular point in the contact surface, with the axis $\theta = 0$ drawn along the contact surface and the z axis in the longitudinal direction (Fig.1).

In each region in the cross section, we have the equilibrium equation

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{1}{r}\tau_{rz} = 0; \tag{1.1}$$

and the following relations between the stress, strain, and displacement components:

$$\tau_{rz} = 2\frac{\sigma_0}{\varepsilon_0}\gamma_{rz},\ \ \tau_{\theta z} = 2\frac{\sigma_0}{\varepsilon_0}\gamma_{\theta z},\ \ 2\gamma_{rz} = \frac{\partial w}{\partial r},\ 2\gamma_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial \theta}. \tag{1.2}$$

Here σ_0 and ε_0 are the stress and strain intensities:

$$\sigma_0 = \sqrt{\tau_{rz}^2 + \tau_{\theta z}^2},\ \ \varepsilon_0 = \sqrt{\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2}.$$

We assume the following relationship between these intensities

$$\sigma_0 = k\varepsilon_0^m,\ \ k = k(\theta),\ \ 0 \leqslant m \leqslant 1, \tag{1.3}$$

in which $k(\theta)$ characterizes the inhomogeneous deformation features of the materials and is to be determined by appropriate experiment. We take the degrees of hardening m as identical for the two materials, while the $k(\theta)$ are different.

We exclude the stress components from (1.1)-(1.3) to get a differential equation for w:

$$\frac{\partial}{\partial r}\left(rk\varepsilon_0^{m-1}\frac{\partial w}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{k}{r}\varepsilon_0^{m-1}\frac{\partial w}{\partial \theta}\right) = 0. \tag{1.4}$$

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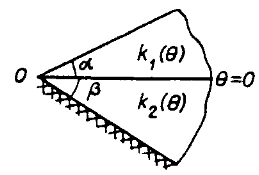

Fig. 1

We assume the following boundary conditions:

$$\tau_{\theta z} = 0 \quad \text{for} \quad \theta = \alpha, \quad w = 0 \quad \text{for} \quad \theta = -\beta. \tag{1.5}$$

We assume that the displacement does not change sign and construct the solution in two regions: $0 \le \theta \le \alpha$ and $\beta \le \theta \le 0$, whose values are denoted by the subscripts i = 1, 2 correspondingly.

We give the following form to the stress and displacement components in each region:

$$\tau_{rzi} = k_i\lambda r^{(\lambda-1)m}\chi_i f_i, \quad \tau_{\theta zi} = k_i r^{(\lambda-1)m}\chi_i f_i',$$
$$w_i = r^\lambda f_i, \quad \chi_i = (\sqrt{f_i'^2 + \lambda^2 f_i^2})^{m-1}. \tag{1.6}$$

Here $f_i = f_i(\theta, \lambda)$ and λ are the unknown functions and a constant.

We substitute the expressions for the displacements from (1.6) into (1.4) to get the differential equation

$$(k_i f_i'\chi_i)' + \eta k_i f_i \chi_i = 0, \quad \eta = \lambda[1 + (\lambda - 1)m]. \tag{1.7}$$

The (1.5) conditions and the (1.6) representation give the boundary conditions

$$f_1'(\alpha) = f_2(-\beta) = 0, \tag{1.8}$$

and also the linkage conditions at the contact surface

$$f_1 = f_2, \quad f_1'\chi_1 = \gamma f_2'\chi_2 \quad \text{for} \quad \theta = 0 \quad (\gamma = k_2(0)/k_1(0)). \tag{1.9}$$

We introduce the new function $\psi_i(\theta, \lambda)$ as

$$f_i' = f_i\psi_i, \tag{1.10}$$

to get a differential equation from (1.7)-(1.9):

$$\psi_i' = -\frac{(\psi_i^2 + \lambda^2)(\psi_i^2 + 2nh_i\psi_i + s^2)}{\psi_i^2 + \lambda^2 n}$$
$$(2h_i = k_i'/k_i, \ n = 1/m, \ s^2 = \lambda(\lambda + n - 1)) \tag{1.11}$$

with the boundary conditions

$$\psi_1(\alpha) = 0, \quad \psi_2(-\beta) = \infty \tag{1.12}$$

and the condition at the contact surface

$$\mu_1(\sqrt{\mu_1^2 + \lambda^2})^{m-1} = \gamma\mu_2(\sqrt{\mu_2^2 + \lambda^2})^{m-1}$$
$$(\mu_i = \psi_i(0, \lambda)). \tag{1.13}$$

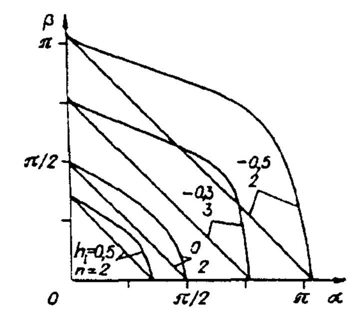

Fig. 2

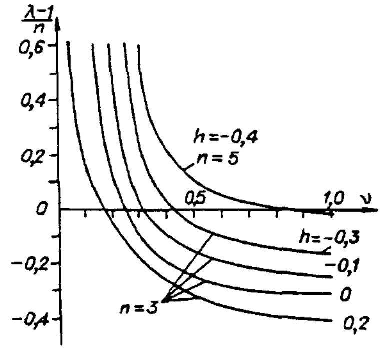

Fig. 3

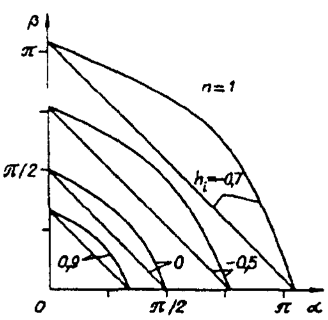

Fig. 4

If there is exponential inhomogeneity, i.e.,

$$k_i = k_i^* e^{2h_i^*\theta} \tag{1.14}$$

(k_i^* and h_i^* are constants of the materials), then h_i = const.

Then the general solution to (1.11) with $\Delta_i = s^2 - n^2h_i^2 > 0$ is put as

$$\frac{E_i}{\sqrt{\Delta_i}}\operatorname{arctg}\frac{\psi_i + nh_i}{\sqrt{\Delta_i}} + G_i\operatorname{arctg}\frac{\psi_i}{\lambda} + \\ + Q_i\ln\left(\sqrt{\frac{\psi_i^2 + 2nh_i\psi_i + s^2}{\psi_i^2 + \lambda^2}}\,\frac{\lambda}{s}\right) = H_i - \theta, \tag{1.15}$$

in which H_i are arbitrary constants. The symbols here are

$$B_iG_i = (n-1)^2,\ B_iE_i = 2(n+1)n^2h_i^2 + (1-\lambda)(n-1)^2, \\ B_iQ_i = 2(n-1)nh_i,\ B_i = (n-1)^2 + 4n^2h_i^2. \tag{1.16}$$

We use the (1.12) boundary conditions to get from (1.15) for the various ranges in θ that

$$H_1 = \alpha + \frac{E_1}{\sqrt{\Delta_1}}\operatorname{arctg}\frac{nh_1}{\sqrt{\Delta_1}},$$

$$H_2 = -\beta + \left(\frac{E_2}{\sqrt{\Delta_2}} + G_2\right)\frac{\pi}{2} + Q_2\ln\frac{\lambda}{s}.$$

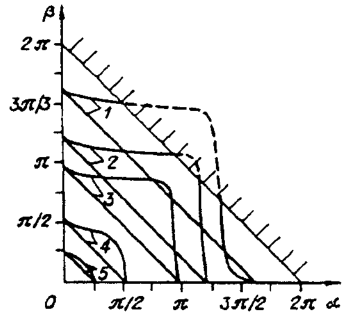

Fig. 5

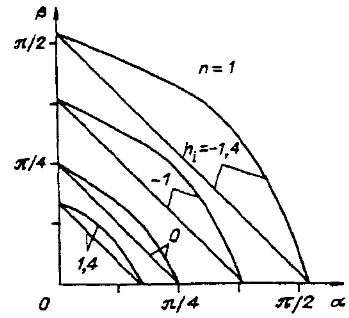

Fig. 6

We take $\theta = 0$ in (1.15) to get

$$\alpha = \frac{E_1}{\sqrt{\Delta_1}}\left(\operatorname{arctg}\frac{\mu_1 + nh_1}{\sqrt{\Delta_1}} - \operatorname{arctg}\frac{nh_1}{\sqrt{\Delta_1}}\right) + G_1 \operatorname{arctg}\frac{\mu_1}{\lambda} + Q_1 \ln\left(\sqrt{\frac{\mu_1^2 + 2nh_1\mu_1 + s^2}{\mu_1^2 + \lambda^2}}\,\frac{\lambda}{s}\right),$$

$$\beta = \frac{E_2}{\sqrt{\Delta_2}}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{\mu_2 + nh_2}{\sqrt{\Delta_2}}\right) + G_2\left(\frac{\pi}{2} - \operatorname{arctg}\frac{\mu_2}{\lambda}\right) - \frac{Q_2}{2}\ln\frac{\mu_2^2 + 2nh_2\mu_2 + s^2}{\mu_2^2 + \lambda^2}. \tag{1.17}$$

Then (1.13) and (1.17) constitute a system of three transcendental equations for μ_1, μ_2, and λ, which ultimately determines $\lambda = \lambda(\alpha, \beta, \gamma, nh_i)$.

The condition $\lambda = 1$ in the space of the parameters α, β, γ, n, and h_i defines a certain finite-stress surface separating the low-stress zone from the zone of high stress concentration. We put $\lambda = 1$ to get from (1.13) and (1.17) that

$$\mu_1\left(\sqrt{\mu_1^2 + 1}\right)^{m-1} = \gamma\mu_2\left(\sqrt{\mu_2^2 + 1}\right)^{m-1}, \quad h_i^2 < m,$$

$$\alpha = \frac{E_1}{\sqrt{\Delta_1}}\left(\operatorname{arctg}\frac{\mu_1 + nh_1}{\sqrt{\Delta_1}} - \operatorname{arctg}\frac{nh_1}{\sqrt{\Delta_1}}\right) + G_1 \operatorname{arctg}\mu_1 + \frac{Q_1}{2}\ln\frac{\mu_1^2 + 2nh_1\mu_1 + n}{n(\mu_1^2 + 1)}, \tag{1.18}$$

$$\beta = \frac{E_2}{\sqrt{\Delta_2}}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{\mu_2 + nh_2}{\sqrt{\Delta_2}}\right) + G_2\left(\frac{\pi}{2} - \operatorname{arctg}\mu_2\right) - \frac{Q_2}{2}\ln\frac{\mu_2^2 + 2nh_2\mu_2 + n}{\mu_2^2 + 1}.$$

Here $B_iE_i = 2(n + 1)n^2h_i^2$, $\Delta_i = n - n^2h_i^2$, while the values of G_i, B_i, and Q_i remain as in (1.16).

Figure 2 shows numerical results from the transcendental system (1.18) in the $\alpha\beta$ plane, where we show the change in the low-stress zone (below the curves) in relation to the inhomogeneity in the mechanical properties for a compound inhomogeneous wedge ($\gamma \neq 1$) and for a continuous one ($\gamma = 1$, $h_1 = h_2$). The straight lines correspond to $\gamma = 1$ and the curves to $\gamma = 2$.

A. Single Inhomogeneous Wedge. When the wedge is made from a single inhomogeneous material, i.e., for $\gamma = 1$, $h_1 = h_2 = h$, we put $\mu_1 = \mu_2$ to satisfy (1.13) identically, while (1.17) gives

$$\alpha + \beta = \left(\frac{E}{\sqrt{\Delta}} + G\right)\frac{\pi}{2} - \frac{E}{\sqrt{\Delta}}\operatorname{arctg}\frac{nh}{\sqrt{\Delta}} + Q\ln\frac{\lambda}{s}. \tag{1.19}$$

We introduce the symbols $\alpha + \beta = 2\alpha_*$, $\nu = \alpha_*/\pi$ to derive a transcendental equation for λ from (1.19):

$$\nu + \frac{E}{2\pi\sqrt{\Delta}}\operatorname{arctg}\frac{nh}{\sqrt{\Delta}} - \frac{Q}{2\pi}\ln\frac{\lambda}{s} - \frac{1}{4}\left(\frac{E}{\sqrt{\Delta}} + G\right) = 0. \tag{1.20}$$

We solve (1.20) numerically to define λ as a function of ν, n, and h. Figure 3 shows a family of $(\lambda - 1)/n$ curves for various n and h with ν as independent variable. For the same degree of hardening and wedge angle, the edge may be in a state of low strain ($\lambda > 1$) or in a state of high stress concentration ($\lambda < 1$) in accordance with the inhomogeneity in the mechanical parameters.

If there is stress concentration at the vertex for a solid homogeneous wedge with vertex angle greater than $\pi/2$ in all cases, while there is none such with an angle less than $\pi/2$, then for a solid inhomogeneous wedge this is not so, as Figs. 2 and 3 show.

B. Linearly Elastic Inhomogeneous Compound Wedge. When the compound wedge is made from linearly elastic inhomogeneous material, we take m = 1 in (1.13) and (1.17) to get an equation for λ:

$$\frac{h_1 + \sqrt{\lambda^2 - h_1^2}\,\mathrm{tg}\,(\alpha\sqrt{\lambda^2 - h_1^2})}{\sqrt{\lambda^2 - h_1^2} - h_1\,\mathrm{tg}\,(\alpha\sqrt{\lambda^2 - h_1^2})}\sqrt{\lambda^2 - h_1^2} - \gamma\sqrt{\lambda^2 - h_2^2}\,\mathrm{ctg}\,(\beta\sqrt{\lambda^2 - h_2^2}) - h_1 + \gamma h_2 = 0. \tag{1.21}$$

Further, we assume λ = 1 to get the equation for the limiting low-stress curves:

$$\frac{h_1 + \sqrt{1 - h_1^2}\,\mathrm{tg}\,(\alpha\sqrt{1 - h_1^2})}{\sqrt{1 - h_1^2} - h_1\,\mathrm{tg}\,(\alpha\sqrt{1 - h_1^2})}\sqrt{1 - h_1^2} - \gamma\sqrt{1 - h_2^2}\,\mathrm{ctg}\,(\beta\sqrt{1 - h_2^2}) - h_1 + \gamma h_2 = 0. \tag{1.22}$$

Here the condition $|h_i| < 1$ is obeyed. The boundary curves defined by (1.22) are shown in Fig. 4 for various h_i, where the straight lines correspond to $\gamma = 1$ and the curves to $\gamma = 2$. This shows that the low-stress zones for an inhomogeneous wedge may be enlarged or diminished by comparison with a homogeneous wedge in accordance with the degree of inhomogeneity.

We get the corresponding formulas in [1, 3] from these results for homogeneous wedges with $h_i = 0$.

2. Planar Strain. We now consider the low-stress state for a compound wedge made of inhomogeneous incompressible materials showing power-law hardening, which is in a state of planar strain. Here we use the Fig. 1 scheme. We assume that the edge $\theta = \alpha$ is free from load, while the edge $\theta = -\beta$ is rigidly gripped.

The following differential equations of equilibrium apply in each wedge region:

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{2}{r}\tau_{r\theta} = 0, \tag{2.1}$$

and the relations between the strain and displacement components are

$$\varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial\theta}, \quad 2\gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial\theta},$$

and those between the stress and strain components are

$$\sigma_r - \sigma = 2\frac{\sigma_0}{\varepsilon_0}(\varepsilon_r - \varepsilon), \quad \sigma_\theta - \sigma = 2\frac{\sigma_0}{\varepsilon_0}(\varepsilon_\theta - \varepsilon),$$

$$\tau_{r\theta} = 2\frac{\sigma_0}{\varepsilon_0}\gamma_{r\theta}, \quad \sigma = \frac{1}{2}(\sigma_r + \sigma_\theta).$$

Here

$$\sigma_0 = \frac{1}{2}\sqrt{(\sigma_r - \sigma_\theta)^2 + \tau_{r\theta}^2}, \quad \varepsilon_0 = \sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + 4\gamma_{r\theta}^2}$$

are the intensities of the stresses and strains, for which we assume (1.3) with identical m for the two materials but differing $k(\theta)$.

We assume that the material is incompressible ($\varepsilon = 0$) for each region, i.e.,

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} = 0. \tag{2.2}$$

The quantities in the regions $0 \le \theta \le \alpha$, $-\beta \le \theta \le 0$ are denoted correspondingly by the subscripts i = 1, 2.

A. Case $\lambda \neq 1$. The displacement pattern in each region that satisfies the incompressibility condition (2.2) is put as

$$u_i = r^\lambda f_i', \quad v_i = -(\lambda + 1) r^\lambda f_i, \quad w_i = 0,$$

in which $f_i = f_i(\theta, \lambda)$ and λ are the unknown eigenfunctions and eigenvalue. The stress components are

$$\sigma_{ri} = \sigma_{\theta i} + 4\lambda k_i r^{(\lambda-1)m} f_i'\chi_i\,, \quad \tau_{r\theta i} = k_i r^{(\lambda-1)m}[f_i'' + (1 - \lambda^2)f_i]\chi_i. \tag{2.3}$$

Here

$$\chi_i = \{\sqrt{[f_i'' + (1 - \lambda^2)f_i]^2 + 4\lambda^2 f_i'^2}\}^{m-1}.$$

We substitute (2.3) into the equilibrium equations (2.1) to get

$$\sigma_{\theta i} = -\frac{r^{(\lambda-1)m}}{(\lambda - 1)m}\{(k_i[f_i'' + (1 - \lambda^2)f_i]\chi_i)' + 4\eta k_i f_i'\chi_i\}, \quad \lambda \neq 1$$

and the differential-equation system

$$\{k_i[f_i'' + (1 - \lambda^2)f_i]\chi_i\}'' + k_i\left(1 - \frac{\eta^2}{\lambda^2}\right)[f_i'' + (1 - \lambda^2)f_i]\chi_i + \\ + 4\eta(k_i f_i'\chi_i)' = 0, \quad \eta = \lambda[1 + (\lambda - 1)m]. \tag{2.4}$$

The boundary conditions at the outer surfaces of the wedge are

$$f_2' = f_2 = 0 \ \text{for} \ \theta = -\beta, \\ \{k_1[f_1'' + (1 - \lambda^2)f_1]\chi_1\}' + 4\eta k_1 f_1'\chi_1 = 0, \\ f_1'' + (1 - \lambda^2)f_1 = 0 \ \text{for} \ \theta = \alpha; \tag{2.5}$$

and at the contact surface

$$[f_1'' + (1 - \lambda^2)f_1]\chi_1 = \gamma[f_2'' + (1 - \lambda^2)f_2]\chi_2, \quad \gamma = k_2(0)/k_1(0), \\ \{k_1[f_1'' + (1 - \lambda^2)f_1]\chi_1\}' + 4\eta k_1 f_1'\chi_1 = \\ = \{k_2[f_2'' + (1 - \lambda^2)f_2]\chi_2\}' + 4\eta k_2 f_2'\chi_2, \\ f_1 = f_2, \ f_1' = f_2' \ \text{for} \ \theta = 0. \tag{2.6}$$

System (2.4) with (2.5) and (2.6) is a three-point problem on eigenvalues for $f_i(\theta, \lambda)$ and λ.

In a semi-inverse fashion, we assign various values to $\lambda = \lambda_* < 1$ to derive numerically from (2.4)-(2.6) the relation between the parameters α, β, γ, and m and the materials inhomogeneity parameters for a given stress concentration. With $\lambda = \lambda_* > 1$, we determine the low-stress region in the space of these parameters.

When the (1.10) substitution is made, the order of the (2.4) equation is reduced along with the boundary conditions (2.5) and (2.6).

B. Case $\lambda = 1$. The finite-stress case requires special examination. The displacement pattern satisfying (2.2) is put as

$$u_i = rf_i', \quad v_i = -2rf_i + C_i r\ln r, \quad w_i = 0.$$

Here $f_i = f_i(\theta)$ and C_i are arbitrary functions and constants. We represent the stress components as

$$\sigma_{ri} = \sigma_{\theta i} + 4k_i\psi_i\chi_i, \quad \tau_{r\theta i} = k_i(\psi_i' + C_i)\chi_i \\ (\chi_i = [\sqrt{4\psi_i^2 + (\psi_i' + C_i)^2}]^{m-1}, \quad \psi_i = f_i')$$

and substitute into (2.1) to get

$$\sigma_{\theta i} = E_i - D_i \ln r - 2\int_0^\theta k_i(\psi_i' + C_i)\chi_i d\theta$$

(E_i and D_i are arbitrary constants) with the differential equation

$$[k_i(\psi_i' + C_i)\chi_i]' + 4k_i\psi_i\chi_i = D_i. \tag{2.7}$$

We use the boundary conditions $\sigma_{\theta 1} = 0$ at $\theta = \alpha$ and $v_2 = 0$ at $\theta = -\beta$ together with the continuity conditions for $\sigma_{\theta i}$ and v_i at the to get

$$C_i = D_i = 0,\ E_i = 2\int_0^\alpha k_1\psi_1'\chi_1 d\theta,\ f_1(0) = f_2(0),\ f_2(-\beta) = 0,$$
$$\sigma_{\theta 1} = 2\int_\theta^\alpha k_1\psi_1'\chi_1 d\theta,\ \sigma_{\theta 2} = 2\left(\int_0^\alpha k_1\psi_1'\chi_1 d\theta + \int_\theta^0 k_2\psi_2'\chi_2 d\theta\right). \tag{2.8}$$

The other boundary conditions give

$$\psi_1'(\alpha) = \psi_2(-\beta) = 0,\ \psi_1 = \psi_2,\ \psi_1'\chi_1 = \gamma\psi_2'\chi_2 \ \text{ for } \ \theta = 0. \tag{2.9}$$

Certain transformations give from (2.7) that

$$(\psi_i'' + 4\psi_i)\frac{m\psi_i'^2 + 4\psi_i^2}{\psi_i'^2 + 4\psi_i^2} + \frac{k_i'}{k_i}\psi_i' = 0, \tag{2.10}$$

whence we introduce the new function $\psi_i = \psi_i'/\psi_i$, to get a first-order equation system

$$\varphi_i' = -\frac{k_i'}{k_i}\frac{\varphi_i^2 + 4}{m\varphi_i^2 + 4}\varphi_i - \varphi_i^2 - 4 \tag{2.11}$$

with the boundary conditions

$$\varphi_1(\alpha) = 0,\ \varphi_2(-\beta) = \infty,$$
$$\varphi_1(\sqrt{4 + \varphi_1^2})^{m-1} = \gamma\varphi_2(\sqrt{4 + \varphi_2^2})^{m-1} \ \text{ for } \ \theta = 0. \tag{2.12}$$

Then (2.11) with (2.12) defines a finite-stress hypersurface that incorporates the inhomogeneity and the physical nonlinearity.

The exponential inhomogeneity in (1.14) has been used in a numerical solution to the (2.9) and (2.10) boundary-value problem, which gives $\beta = \beta(\alpha, n, \gamma, h_i)$, where $n = 1/m$, $h_i = k_i'/k_i = \text{const}$, $\gamma = k_2(0)/k_1(0)$. Equation (2.11) and (2.12) are inconvenient for numerical solution. Figure 5 shows the numerical results, which imply that the low-stress zones alter considerably with h. The straight lines correspond to $\gamma = 1$ and the curves to $\gamma = 2$. Lines 1-5 correspond to the following parameters: $h_i = -1.36$, $n = 2$; $h_i = -0.85$, $n = 5$; $h_i = -1$, $n = 3$; $h_i = -1$, $n = 2$; $h_i = 0$, $n = 2$. The dashed parts of the lines correspond to those cases inhomogeneity when finite stresses may be absent at the tip of the semi-infinite slot ($\alpha + \beta = 2\pi$).

C. Linearly Elastic Inhomogeneous Compound Wedge. If a compound wedge is made of linearly elastic inhomogeneous materials, we use (2.11) and (2.12) with $m = 1$ and the exponential inhomogeneity (1.14) with $|h_i| < 2$ to get the transcendental equation

$$\frac{4\operatorname{tg}(\alpha\sqrt{4 - h_1^2})}{\sqrt{4 - h_1^2} - h_1 \operatorname{tg}(\alpha\sqrt{4 - h_1^2})}$$
$$- \gamma\sqrt{4 - h_2^2}\operatorname{ctg}(\beta\sqrt{4 - h_2^2}) + \gamma h_2 = 0. \tag{2.13}$$

We take $h_i = 0$ for a homogeneous compound linearly elastic wedge and get from (2.13) the corresponding equation of [3] if one takes the Poisson's ratios for the materials in it as 1/2.

Figure 6 shows the traces of the surface defined by (2.13) in the $\alpha\beta$ plane (the straight lines correspond to $\gamma = 1$ and the curves to $\gamma = 2$).

There are always stress concentrations at the vertex for a solid homogeneous wedge with vertex angle greater than $\pi/4$, whereas there are none for an angle less than $\pi/4$, but that regularity is violated for a solid inhomogeneous wedge, as Figs. 5 and 6 show.

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